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# Edit languages for information trees

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**Abstract:** We consider a simple set of edit operations for unordered, edge-labeled trees, called *information trees* by Dal Zilio *et al* [DLM04]. We define *tree languages* using the *sheaves automata* from [FPS07] which in turn are based on [DLM04] and provide an algorithm for deciding whether a complex edit preserves membership in a tree language. This allows us to view sheaves automata and subsets of tree edits as *edit languages* in the sense of [HPW12]. They can then be used to instantiate the framework of *edit lenses* between such languages and model concrete examples such as synchronisation between different file systems or address directories.

**Keywords:** Bidirectional programming, lens, edit, information tree

## 1 Introduction

Semantic models of bidirectional transformations are generally presented as transformations between the *states* of replicas. For example, the familiar framework of asymmetric lenses defines a lens between replicas of types $A$ and $B$ as a pair of a *get* function from $A$ to $B$ and a *put* function from $A \times B$ to $A$. An implementation directly based on this semantics would pass to the *put* function the entire states of the original $A$ and updated $B$ replicas.

Though pleasingly simple, this treatment falls short of telling a full story in at least two important ways. First, it does not explain how the lens should *align* the parts of the old $A$ and the new $B$ so that the parts of $A$ that are "hidden" in the $B$ view retain their positions in the result. For example, if $A$ and $B$ are both lists of people where each element of $A$ includes a name, address, and email while the corresponding elements of $B$ give only a name and address, the user will reasonably expect that inserting a new element at the head of the $B$ replica will lead to an updated $A$ replica where each existing name keeps its associated email. And second, the simple "classical" form of asymmetric lenses fails to capture the reasonable expectation that a small change to the $B$ replica can be transformed into a change to the $A$ replica using time and space proportional to the size of the change, not to the sizes of the replicas.

One way to address at least the first concern is to enrich the basic structure of a state-based lens with a new input to the *put* function, a data structure that explicitly represents the *alignment* between the original and updated $B$ replicas; this idea forms the basis for *dictionary lenses* [BFP+08], *matching lenses* [BCF+10], and *symmetric* [DXC+11b] and *asymmetric delta lenses* [DXC11a] (based on [Ste07]). Another approach is to annotate the $B$ structures themselves with change information [XLH+07, HHI+10]. However, all these approaches still involve whole replica states, either as explicit inputs and outputs of the *put* function or implicitly as part of the type to which a delta belongs. Thus, it is not clear whether these models can be implemented in such a way that small changes to a replica are propagated in time and space proportional to the size of the change.

In earlier work [HPW12], we proposed going a step further and define lenses that work exclusively with edits. We defined a semantic model called *edit lenses* in which the sets of source and target replicas $A$ and $B$ are enriched with monoids of *edits* and a lens's *get* and *put* functions map edits to edits. This work was carried out in an abstract algebraic setting where the actual data structures being transformed and the exact shapes of edits to them were left unspecified; in this setting, we showed how a number of familiar constructions on lenses—products, sums, etc.—could be carried out.

The present paper[1] takes a first step toward instantiating this abstract semantic model with concrete data structures and a concrete notion of edits. The data model we choose is a very common and expressive one: unordered, edge-labeled trees—called *information trees* by Dal Zilio *et al* [DLM04]. These can encode a variety of data formats, including XML-style trees, their original application. Furthermore, trees have been used to represent graphs [BDHS96, HHI+10] by unrolling up to bisimulation. This paper thus makes a first step towards general edit languages for trees. Our edit operations comprise in particular insertion and deletion of subtrees and renaming of edges; we show how these give rise to edit languages on tree languages and can thus be used to instantiate the framework of edit lenses so as to yield bidirectional synchronization between information trees.

Our main technical result is that weakest preconditions of our edits can be efficiently computed for sets of information trees specified by sheaves automata. This allows for efficient "type checking" of edits and thus permits automatic checks that an edit language for trees presented as a sheaves automaton and certain sequences of atomic edits is indeed well-formed.

The present work is thus a first step; we hope that the introduction of tree automata into the world of editing and synchronization will also lead to high-level support for constructing edit lenses themselves and for checking their soundness, but this remains future work.

## 2 Trees and sheaves automata

This section introduces some notations and definitions that we need in the sequel. Some of the descriptions are taken from [FPS07].

### 2.1 Trees

We assume a finite alphabet $\Sigma$ and consider unordered trees whose edges are labeled by $\Sigma^*$. We write trees with a pair of braces $\{|\}$ for each node; each subtree is written $\mathtt{n} \mapsto t$, where $\mathtt{n}$ names the edge that leads to the subtree $t$. To reduce clutter, we abbreviate the tree $\{|\mathtt{n} \mapsto \{|\}|\}$ to just $\mathtt{n}$ when no confusion arises. For example, here is an explicit tree with two children labeled "name" and "email"...

$$\{|\mathtt{name} \mapsto \{|\mathtt{John} \mapsto \{|\}|\}, \mathtt{email} \mapsto \{|\mathtt{john@example.com} \mapsto \{|\}|\}|\}$$

...and its abbreviated form:

$$\{|\mathtt{name} \mapsto \mathtt{John}, \mathtt{email} \mapsto \mathtt{john@example.com}|\}$$

[1] An expanded version with additional examples and explanation is available at `http://dmwit.com/papers/201302ELfIT_full.pdf`.

We write $t(n)\downarrow$ to indicate that tree $t$ has child $n$, $t(n)\uparrow$ to indicate that tree $t$ does not have child $n$, and $t(n)$ for the child under the edge labeled $n$. We write $dom(t) = \{n \mid n \in \Sigma^* \wedge t(n)\downarrow\}$. The expression $t[n \mapsto t']$ describes the tree that agrees with $t$ everywhere, except that its child under label $n$ is $t'$. The expression $t \setminus \{n_1, n_2, \ldots, n_k\}$ describes the tree that is undefined at $n_1, n_2, \ldots, n_k$ but agrees with $t$ everywhere else. When context makes it clear that $n$ is a name, we write $t \setminus n$ to mean $t \setminus \{n\}$. To simplify the definition of partial functions, we take an expression like $t[n \mapsto E]$ when $E$ is undefined to be undefined itself.

## 2.2 Tree edits

The set of *atomic tree edits* is defined as follows (where $e$ ranges over edits, $t$ over trees, and $m$ and $n$ over names):

$$e \;::=\; insert(t) \mid hoist(m,n) \mid delete(m) \mid rename(m,n) \mid at(n,e)$$

The application of an atomic edit $e$ to a tree $t$ is either undefined ($\perp$) or a new tree defined as follows:

$$\begin{array}{rll}
insert(t') \cdot t = t \cup t' & & \text{if } dom(t) \cap dom(t') = \emptyset \\
hoist(m,n) \cdot t = t[m \mapsto t(m) \setminus n, n \mapsto t(m)(n)] & & t(m)(n)\downarrow \wedge\, t(n)\uparrow \\
delete(n) \cdot t = t \setminus n & & t(n) = \{\!|\}\!| \\
rename(n,n') \cdot t = (t \setminus n)[n' \mapsto t(n)] & & t(n)\downarrow \wedge\, t(n')\uparrow \\
at(n,e) \cdot t = t[n \mapsto e \cdot t(n)] & & t(n)\downarrow \\
e \cdot t = \perp & & \text{in all other cases}
\end{array}$$

Tree edits are sequences of atomic edits. We overload $\cdot$ as tree edit application, which is the natural lifting of atomit edit application to sequences:

$$\begin{array}{r}
\langle\rangle \cdot t = t \\
\langle a_1, \ldots, a_n \rangle \cdot t = \langle a_1, \ldots, a_{n-1} \rangle \cdot (a_n \cdot t)
\end{array}$$

## 2.3 Sheaves formulae and automata

Intuitively, a *sheaves automaton* consists of a set of states, each associated with a *sheaves formula*; each sheaves formula describes a set of trees by specifying which names may occur as immediate child edges and, for each one, an automaton state that describes the subtree found beneath it.

To describe sheaves automata more formally, we must first fix a formalism for writing down arithmetic constraints. We use Presburger arithmetic—the decidable first-order theory of the naturals with addition but without multiplication—for this purpose[2]. Expressions in Presburger

[2] Here we follow the lead of [FPS07] and [DLM04]. Presburger arithmetic is a particularly expressive logic, and it is possible that a simpler fragment suffices, but we leave an investigation of this possibility to future work.

arithmetic include constants, variables, and sums, and formulae include equalities between expressions, boolean combinations of formulae, and quantified formulae:

$$
\begin{aligned}
m &::= 0, 1, 2, \ldots \\
v &::= m \mid x_i \mid v + v \\
\phi &::= v_1 = v_2 \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \exists\phi
\end{aligned}
$$

We use a de Bruijn representation—a variable $x_j$ within the scope of $k$ quantifiers represents the $(j-k)$th free variable if $j \geq k$, and otherwise is bound by the $j$th enclosing quantifier, counting from the inside-out.

The semantics of a Presburger formula is the set of vectors of naturals that satisfy it. We write $\bar{c} \vDash \phi$ for formula satisfaction, substituting $c_i$ for $x_i$, and $fv(\phi)$ for the set of free variables in $\phi$.

Next, a *sheaves automaton* comprises a finite set of states together with a mapping $\Gamma$ from states to *sheaves formulae*. The transition behavior from a state is given by the sheaves formula associated with it in $\Gamma$. Each sheaves formula has two components—a Presburger formula $\phi$ and a list of *elements*, each of the form $r_i[s_i]$, where $r_i$ is a regular language called the *tag* of the element, and $s_i$ is a state. The operation of a sheaves automaton is like a bottom-up regular tree automaton. Let $t$ be a tree and $s$ be an automaton state with $\Gamma(s) = (\phi, [r_0[s_0], \ldots, r_k[s_k]])$. For each $i$ in the range 0 to $k$, let $c_i$ be the number of children $n \in dom(t)$ for which $n \in r_i$ and $t(n)$ is accepted by $s_i$. Then $t$ is accepted by $s$ iff the vector $(c_0, \ldots, c_k) \vDash \phi$.

Note that the integers that represent variables in de Bruijn notation give the correspondence between free variables in $\phi$ and elements—the constraint on $x_i$ controls the number of children whose name matches $r_i$ with subtrees accepted by $s_i$.

In the following, it will sometimes be convenient to treat elements (and the corresponding Presburger variables in the sheaves formula) as if they were indexed by some set with more structure than the natural numbers. This is perfectly reasonable, provided there is an injective mapping from the structured set to the naturals. When it is clear that such an injective mapping exists (and in particular especially when the structured set is finite) we will leave the mapping unspecified and simply use values from the structured set as subscripts to the collections $\bar{r}$ and $\bar{s}$ as well as any Presburger variables, understanding $r_v$ to stand for $r_{f(v)}$.

Sheaves automata and sheaves formulae are subject to some well-formedness conditions. A sheaves formula $(\phi, E)$ with $|E| = k$ is well-formed iff the free variables of $\phi$ are $\{x_0, \ldots, x_{k-1}\}$; the elements are pairwise disjoint—i.e., if the list includes $r_i[s_i]$ and $r_j[s_j]$ and there exists a tree accepted by both $s_i$ and $s_j$, then the regular languages denoted by $r_i$ and $r_j$ are disjoint; and the elements are generating—i.e., for every tree $t$ and label $n \in dom(t)$ there is an element $r_i[s_i]$ such that $n \in r_i$ and $t(n)$ is accepted by $s_i$. A list of elements obeying these conditions is called a *basis*. A sheaves automaton is well-formed iff every sheaves formula in the range of $\Gamma$ is well-formed. These well-formedness conditions guarantee two properties. First, because the elements are non-overlapping, every tree has a unique decomposition over the basis, which means that the semantics of a sheaves automaton is well-defined. Second, because the elements generate the set of all tree slices, certain constructions are simple. For example, $(\phi, E)$ and $(\overline{\phi}, E)$ accept complementary sets of trees.

As an example, the set of trees

$$\{\{\!|\}\!\},\ \{\!|\mathtt{a}, \mathtt{b}|\!\}\}$$

is described by the automaton state $s$, where $\Gamma(s)$ is

$$\left( \begin{array}{l} ((x_0 = 0 \wedge x_1 = 0) \vee (x_0 = 1 \wedge x_1 = 1)) \wedge (x_2 = 0), \\ \left[\mathtt{a}[\top], \mathtt{b}[\top], \overline{\{\mathtt{a},\mathtt{b}\}}[\top]\right] \end{array} \right)$$

and $\top$ is a state that accepts any tree[3]. To see this, observe that the constraints on $x_0$ and $x_1$ force the number of children described by the elements $\mathtt{a}[\top]$ and $\mathtt{b}[\top]$ to both be 0 or both be 1, and that the constraint on $x_2$ forces the number of children belonging to the final element to be 0, that is, there are no edge labels other than `a` or `b`.

The relation $A, s \vdash t$ tells when automaton $A$ accepts tree $t$ when started at state $s$. We often write sheaves automata as $A = (S, s_0, \Gamma)$, where $s_0$ is a distinguished initial state. We then write $A \Vdash t$ to mean $A, s_0 \vdash t$. We also write $L(A) = \{t \mid A \Vdash t\}$ for the language accepted by automaton $A$.

For the definitions in the next section, a small change of notation is convenient: instead of writing a sheaves formula as a pair $(\phi, E)$ of a presburger formula and a sequence $E = r_1[s_1], \ldots, r_n[s_n]$ of elements, we will often write it as $(\phi, \bar{r}, \bar{s})$, giving the sequence of regular languages for child names and the sequence of subtree states separately.

## 3 Weakest preconditions of edits

So far, we have reviewed a definition for trees and a notion of type-checking for trees, namely, sheaves automata. We have also given a definition for tree edits; what remains is to give a notion of type-checking for tree edits. The question we are interested in is: given a desired target tree type and an edit, how do we create a type of source trees that guarantees arrival at the target type? We will press the sheaves automata into service again, building a "source-type" automaton given a particular edit and "target-type" automaton.

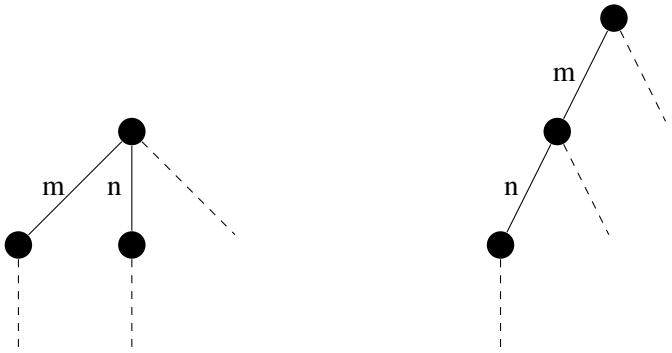
Let $A, B$ be sheaves automata and $e$ be an edit. We write $e : A \to B$ to mean that whenever $A \Vdash t$ and $e \cdot t$ is defined we have $B \Vdash e \cdot t$.

We now define for each edit $e$ and automaton $A$ a new automaton $e \cdot A$ such that $t \in L(e \cdot A)$ iff $e \cdot t \in L(A)$ whenever $e \cdot t$ is defined, that is, $e \cdot A$ is an automaton representing the weakest precondition that ensures that applying edit $t$ will result in a tree of type $A$. Formally: $L(e \cdot A) = \{t \mid e.t \downarrow \Rightarrow e.t \in L(A)\}$. It is then clear that $e : A \to B$ iff $L(A) \subseteq L(e \cdot B)$ (notice that if $e.t$ is undefined then, trivially, $t \in L(e.A)$). Language inclusion of sheaves automata being decidable [DLM04], this then implies a decision procedure for $e : A \to B$ and in particular for deciding whether a given edit belongs to $A \to A$.

**Theorem 1** *Let $A = (S, s_0, \Gamma)$ be a sheaves automaton and $e$ be an atomic tree edit. There exists a sheaves automaton $e \cdot A = (S', s_0', \Gamma')$ such that $t \in L(e \cdot A)$ iff $e \cdot t$ is undefined or $e \cdot t \in L(A)$. Moreover, $e \cdot A$ can be effectively obtained from $e$ and $A$.*

*Proof.* The construction proceeds in two stages. First, we define for each edit $e$ a sheaves automaton $D_e$ such that $L(D_e) = \{t \mid e \cdot t \uparrow\}$.

[3] For example, we could set $\Gamma(\top) = (x_0 = x_0, [\Sigma^*[\top]])$.




(a) The trees automaton $A$ accepts. (b) The trees $hoist(m,n) \cdot A$ should accept.

Figure 1: the *hoist* operation

Then, for each edit $e$, we construct a sheaves automaton $e \star A$ such that for all $t$ with $e \cdot t \downarrow$ one has $e \star A \Vdash t \iff A \Vdash e \cdot t$. We then define the desired automaton $e \cdot A$ so that $L(e \cdot A) = L(D_e) \cup L(e \star A)$ using the union construction from [DLM04].

Given this strategy, the remaining definitions are essentially a programming exercise.

- $e = insert(t')$. For $D_e$ we need to check that the toplevel labels of $t'$ are not present. If these are, say, $r_1 \ldots r_n$ and $r_{n+1}$ is $\Sigma^* \setminus \{r_1, \ldots, r_n\}$ then the sheaves formula $(\sum_{i=1}^{n} x_i \neq 0, \bar{r}, \bar{\top})$ with $\top$ a state accepting any tree achieves the purpose when attached to the initial state of $D_e$.

  To build $e \star A$ we add a fresh state $s_0'$ that is just like $s_0$ except that it has already "seen" the subtree $t'$. That is, if $\Gamma(s_0) = (\phi, \bar{r}, \bar{s})$ and $c_i$ is the number of labels $n \in r_i \cap dom(t')$ for which $A, s_i \vdash t'(n)$, then we define

  $$S' = S \cup \{s_0'\}$$
  $$\phi' = \phi[x_i + c/x_i]$$
  $$\Gamma' = \Gamma[s_0' \mapsto (\phi', \bar{r}, \bar{s})]$$

  where $\phi[e/x]$ is the formula $\phi$ with expression $e$ substituted for variable $x$.

- $e = hoist(m,n)$. Suppose $\Gamma(s_0) = (\phi, \bar{r}, \bar{s})$. The high-level plan for $e \star A$ is to add some new states for each state that the $m$-branch could be accepted under that tell which state the $n$-branch was accepted under. To this end, define sets $I_m$ and $I_n$ so that $m \in r_i$ iff $i \in I_m$ and likewise $n \in r_i$ iff $i \in I_n$. We now perform the following modifications:

  - Add a fresh state $s_0'$, making it the initial state, and letting it initially be a copy of $s_0$.
  - Replace $r_i$ with a regex that matches $L(r_i) \setminus \{n\}$ for each $i \in I_n$.
  - Remove the $I_m$ regexes (and their associated successor states) from the sheaves formula associated with $s_0'$.

- Add the regex that matches exactly $n$, with a fresh successor state $s_n$ which accepts any tree.
- For each $(i, j) \in I_m \times I_n$, add a new regex which is a copy of the original $r_i$ and whose successor state is a copy of the automaton that accepts the language of trees that can be split into an $n$ part accepted by $s_j$ and a remainder accepted by $s_i$. Name the indices of the regexes and successor states added by this operation $k_{ij}$.
- Modify the Presburger formula associated with $s'_0$ to reflect the changes above: for each $i \in I_m$, replace occurrences of $x_i$ with $\sum_j x_{k_{ij}}$, and for each $j \in I_n$, replace occurrences of $x_j$ with $\sum_i x_{k_{ij}}$. (If $i \in I_m \cap I_n$ for some $i$, then these two operations coincide, because of the partitioning property of sheaves formulae.)

The definition of $D_e$ is analogous to the previous case.

- $e = delete(n)$. The automaton that accepts exactly the tree $\{\!|n \mapsto \{\!|\}\!|\}\!|$ is easy to construct; call this automaton $A'$. We then define $D_e = \neg(\top + A')$ using the constructions described in [FPS07]. For $e \star A$, remove $n$ from each of the $r_i$. Then add an extra condition guarded by $n$. Place no constraint on the corresponding cardinality. Leave the other cardinalities as they were.

- $e = rename(n, n')$. Suppose $\Gamma(s_0) = (\phi, \bar{r}, \bar{s})$. Choose a fresh $s'_0$ and define

$$r'_i = \begin{cases} r'_i \cup \{n\} & n' \in r'_i \\ r'_i \setminus \{n\} & n' \notin r'_i \end{cases}$$
$$S' = S \cup \{s_0\}$$
$$\Gamma' = \Gamma[s'_0 \mapsto (\phi, \overline{r'}, \bar{s})]$$

Then this new automata counts any $n$-rooted subtree as if it were an $n'$-rooted one instead.

We should briefly argue that the sheaves formula given for $s'_0$ is generating and pairwise disjoint. It is generating: take any tree $t$ and name $n'' \in dom(t)$, and apply the definition of "generating" to the original sheaves formula using the tree $[n' \mapsto t(n'')]$ if $n'' = n$ or using the tree $[n'' \mapsto t(n'')]$ if not. It is pairwise disjoint: consider $r'_i[s_i]$ and $r'_j[s_j]$ for which both $s_i$ and $s_j$ accept tree $t$. Take any name $n'' = n$ (resp. $n'' \neq n$). Since the original sheaves formula is pairwise disjoint, at most one of $r_i$ and $r_j$ contain $n'$ (resp. $n''$), hence at most one of $r'_i$ and $r'_j$ contain $n''$.

For $D_e$ we use the automaton $(\{\top, s\}, s, \Gamma)$ where:

$$\Gamma(\top) = (0 = 0, [\Sigma^*[\top]])$$
$$\Gamma(s) = (x_0 = 0 \vee x_1 \neq 0, [\{n\}[\top], \{n'\}[\top], \Sigma^* \setminus \{n, n'\}[\top]])$$

- $e = at(n, e')$. Suppose $\Gamma(s_0) = (\phi, \bar{r}, \bar{s})$. Define the set $I$ so that $n \in r_i$ iff $i \in I$. We will apply the edit $e$ to each of the automata that start at $s_i$ such that $i \in I$, and use these modified automata as the new states associated with these regexes. Define:

$$A'_i = e \cdot (S, s_i, \Gamma)$$

Later, we will want to ensure that the $A'_i$ automata are disjoint in the sense that they accept no common trees. This is mostly true of these $A'_i$: since the states $s_i$ accept disjoint trees, the trees which arrive at state $s_i$ after being edited by $e$ are also disjoint sets. However, these $A'_i$ may also accept trees for which the edit $e$ does not apply. Since we don't care what our final automata does with such trees (as we will be dealing with this situation in $D_e$), this subtlty is not important, so we will define it away. The language difference operation can be implemented on automata, so we define $A_i = (S_i, a_i, \Gamma_i) = A'_i \setminus A_{i-1} \setminus \cdots \setminus A_0$.

Additionally, we may define an automata $A_{-1} = (S_{-1}, a_{-1}, \Gamma_{-1})$ which accepts exactly when none of the automata $A_i$ for $i \in I$ do using standard constructions. Without loss of generality, we may assume the $S_i$ are pairwise disjoint and disjoint from $S$. (If not, just rename them—this does not change the behavior of the automata.)

We are now ready to begin constructing the new automaton. We will index the new formula by the set

$$\{\mathsf{edited}(i) \mid i \in I\} \cup \{\mathsf{unedited}(i) \mid 0 \le i < |\bar{r}|\} \cup \{\mathsf{illtyped}\}$$

telling whether we consider the successor state to be descending into an edited child or not. (The illtyped index is used to capture children which would be edited, but for which the edit would not be defined.) We then pick a fresh $s'_0$ and define:

$$\begin{aligned}
S' &= \{s'_0\} \cup S \cup S_{-1} \cup \bigcup_{i \in I} S_i \\
r'_{\mathsf{edited}(i)} &= \{n\} \\
r'_{\mathsf{unedited}(i)} &= r_i \setminus \{n\} \\
r'_{\mathsf{illtyped}} &= \{n\} \\
s'_{\mathsf{edited}(i)} &= a_i \\
s'_{\mathsf{unedited}(i)} &= s_i \\
s'_{\mathsf{illtyped}} &= a_{-1} \\
\rho(x_i) &= x_{\mathsf{edited}(i)} + x_{\mathsf{unedited}(i)} && i \in I \\
\rho(x_i) &= x_{\mathsf{unedited}(i)} && i \notin I \\
\phi' &= \rho\phi \wedge x_{\mathsf{illtyped}} = 0 \\
\Gamma'(s'_0) &= (\phi', \overline{r'}, \overline{s'}) \\
\Gamma'(s) &= \Gamma_i(s) && s \in S_i \\
\Gamma'(s) &= \Gamma(s) && s \in S
\end{aligned}$$

One might worry about whether the states associated with the regular language $\{n\}$ above are disjoint and generating. They are generating because of the addition of the catch-all state $s'_{\mathsf{illtyped}}$, and are disjoint because the $A_i$ are disjoint as automata.

To build $D_e$, we first recursively build $D_{e'} = (S, s_0, \Gamma)$, then create some fresh states $s'_0$, $\neg s_0$, and $\top$. The new automaton will check whether either there is no $n$ child or there is one, but it's accepted by $D'_e$:

$$
\begin{aligned}
S' &= S \cup \{s'_0, \neg s_0, \top\} \\
\Gamma'(\top) &= (0 = 0, [\Sigma^*[\top]]) \\
\Gamma'(s'_0) &= (x_0 \neq 0 \vee x_1 = 0, [\{n\}[s_0], \{n\}[\neg s_0], \Sigma^* \setminus \{n\}[\top]]) \\
\Gamma'(\neg s_0) &= (\neg\phi, e) && \text{where } \Gamma(s_0) = (\phi, e) \\
\Gamma'(s) &= \Gamma(s) && s \in S \\
D_e &= (S', s'_0, \Gamma')
\end{aligned}
$$

□

**Lemma 1** *Language inclusion of sheaves automata is decidable.*

*Proof.* Given sheaves automata $A$ and $B$ to tell whether $L(A) \subseteq L(B)$ build an automaton $C$ such that $L(C) = L(A) \setminus L(B)$ using the algorithms for intersection and complement given in [DLM04]. Then check whether or not $L(C) = \emptyset$. □

**Corollary 1** *Given sheaves automata $A$ and $B$ and edit $e$ it is decidable whether $e : A \to B$.*

*Proof.* Given the theorem, this amounts to deciding whether $L(A) \subseteq L(e \cdot B)$. □

## 4 Edit languages and lenses

In previous work [HPW12] we defined an edit language $E$ as a set $E$ (or $|E|$) of elements, a monoid $\partial E$ of edit operations and a partial action $\cdot : \partial E \times E \to E$. Given a sheaves automaton $A$, we can thus form an edit language $E_A$, the *full tree edit language over* $A$, whose set of elements is $L(A)$ and whose monoid of edits $\partial E_A$ is $\{e \mid e : A \to A\}$. We have seen that membership in $\partial E_A$ is effectively decidable.

For a concrete example of such an edit language consider a file system wherein an edge carrying a label starting with $F$ represents a file, whereas a label starting with $D$ represents a directory. We can easily design a sheaves automaton $FS$ that ensures that all labels are of one of these two kinds and that only directories have children.

The (full) edit monoid $\partial E_{FS}$ then comprises those sequences of atomic edits that preserve this structure and we can effectively check for a given edit that this is the case.

Also recall from [HPW12] that an edit lens between two edit languages $E$ and $E'$ comprises:

- a complement set $C$
- a consistency relation $K \subset |E| \times C \times |E'|$
- a rightward translation $\Rrightarrow \in \partial E \times C \to \partial E' \times C$
- a leftward translation $\Lleftarrow \in \partial E' \times C \to \partial E \times C$

such that:

- consistency and typing are preserved, that is,

$$\frac{(a,c,b) \in K \qquad e \in \partial E \qquad e \cdot a \downarrow \qquad \Rrightarrow (e,c) = (e',c')}{(e \cdot a, c', e' \cdot b) \in K \qquad e' \in \partial E' \qquad e' \cdot b \downarrow}$$

  and similarly for $\Lleftarrow$

- composition of edits is respected, that is,

$$\frac{\Rrightarrow (e_1,c) = (e_1',c') \qquad \Rrightarrow (e_2,c') = (e_2',c'')}{\Rrightarrow (e_2 e_1, c) = (e_2' e_1', c'')}$$

  and similarly for $\Lleftarrow$

It is now possible to design lenses between full tree edit languages of the form $E_A$ for a sheaves automaton $A$, but we believe that this is not necessarily the best way of proceeding, since the edit transformation embodied in the lens might need to get some intensional information about the intended semantics of the edit to be translated. We are thus led to define a *tree edit language* to be an edit language whose set of elements is of the form $|E| = L(A)$ for some sheaves automaton $A$ and whose monoid of edits $\partial E$ comes with a monoid morphism $f : \partial E \to \{e \mid e : A \to A\}$. Thus, every edit is "implemented" as a sequence of atomic edits and, thus, assuming that $\partial E$ is finitely generated, we can check well typedness by checking whether $f(g) : A \to A$ holds for every generator $g$. In the file system example the edit monoid might comprise creation, deletion, and moving of files and directories, each of which can be implemented as a sequence of atomic edits and thus checked to preserve the required structure.

It is then possible to design tree edit lenses that synchronise between different file systems and file structures, for example one having nested subdirectories and the other one being essentially flat. In such a case, the complement will store alignment information in the form of a bijection between files and directories.

## 5 Conclusion

We have defined a simple set of edits for information trees comprising insertions, deletions, relocations, and renamings of subtrees. Our main technical result states that tree languages defined by sheaves automata [DLM04] are effectively closed under weakest preconditions for these edits and that therefore, typechecking of edits against tree types defined by sheaves automata [FPS07] is algorithmically tractable.

We see this result as a first step towards an automata-based high-level formalism for tree synchronisation; in particular we would like to investigate to what extent complements and consistency relations can be defined by automata and how the tree types from [FPS07] and the associated term formers can be lifted to edit lenses. More speculative goals include the automatic discovery of tree edits ("tree diffing") and the extension to graphs.

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